Magooey's Math Problems

Related Rates

Synopsis. Related rates types of problems deal with the velocity or pace of change of a quantity when other data are given. This often includes the rate of change of other variables, and the particular values of those variables. In order to fix ideas, it is best to consider examples of these problems.

Exercises.

1. A rectangle is changing in size. It's length is increasing at a rate of 2 centimeters per second, while its width is decreasing at a rate of 3 centimeters per second. Find the rate of change of the area at the time when the length is 13 and the width is 21.

Solution. We have the equation $A = l \times w$, where A is area, l is length, and w is width. Differentiating we find

$$\frac{dA}{dt} = l \cdot \frac{dw}{dt} + w \cdot \frac{dl}{dt}.$$

We are given $l = 13$, $w = 21$, $dl/dt = 2$ and $dw/dt = -3$. Substituting, we find $dA/dt = 13 \cdot (-3) + 21 \cdot 2 = 3$. So the area is increasing at 3 square centimetes per second. ■

2. A sphere is changing in size. If its radius increases at the speed of 0.25 centimeters per second, how fast is the volume changing when the radius is 5 centimeters? How fast is the surface area changing at the same time?

3. Extruded plastic forms itself into the shape of a rectangular box. The volume of the box is $V = l \times w \times h$ where l is length, w is width and h is height. Assume $dl/dt = 3\,dw/dt$ and $dw/dt = 4\,dh/dt$ How fast is the height changing if plastic is being added at 0.25 cubic centimeters a second, and $l = 0.6$, $w = 0.2$ and $h = 0.05$ centimeters.

Solution. We have $V = l \times w \times h$. Differentiating, using the product rule, we have

$$\begin{aligned}
\frac{dV}{dt} &= l \cdot w \cdot \frac{dh}{dt} + l \cdot \frac{dw}{dt} \cdot h + \frac{dl}{dt} \cdot w \cdot h \\
0.25 &= 0.6 \cdot 0.2 \cdot \frac{dh}{dt} + 0.6 \cdot \frac{dw}{dt} \cdot 0.05 + \frac{dl}{dt} \cdot 0.2 \cdot 0.05 \\
0.25 &= 0.6 \cdot 0.2 \cdot \frac{dh}{dt} + 0.6 \cdot 4\,\frac{dh}{dt} \cdot 0.05 + 12\,\frac{dh}{dt} \cdot 0.2 \cdot 0.05 \\
0.25 &= (0.12 + 0.12 + 0.12)\,\frac{dh}{dt} \\
\frac{25}{36} &= \frac{dh}{dt}.
\end{aligned}$$

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4. A rhombus is changing in size. One side is increasing at a rate of 1 centimeter per second, while the other side is decreasing at a rate of 2 centimeters per second. The angle between the sides is decreasing at a rate of .3 radians per second. How fast is the area changing at the instant when the first side has length 7, the other side has length 5 and the included angle is $\pi/3$ or 60 degrees.

5. A 26 foot tall ladder is slipping down a wall at 4 feet per second. Find the speed at which the bottom of the ladder is moving away from the wall, when the height of the top of the ladder is 24 feet above the floor.

6. A train is moving in a straight line at 80 miles per hour. You are standing 2 miles from the nearest point on the train track. How fast is the distance between you and the train increasing when the train is moving away from you and is 5 miles from the nearest point on the train track.

Solution. Let a be the distance of the train along the track with respect to the nearest point. Let b be the distance between yourself and the train. By the Pythagorean Theorem $a^2 + 2^2 = b^2$. Thus $2\,a \cdot \frac{da}{dt} + 0 = 2\,b \cdot \frac{db}{dt}$. Canceling 2's and substituting, we find $5 \cdot 80 = b \cdot \frac{db}{dt}$. By the Pythagorean Theorem $b = \sqrt{5^2 + 2^2} = \sqrt{29}$. Hence $\frac{db}{dt} = 400\sqrt{29}/29$. ■

7. In the previous train problem, at what rate is the angle between the train, yourself, and the nearest point of the track, increasing?

8. A hopper in the shape of an inverted cone deposits liquid through a tiny hole in the bottom, at the apex of the cone. The rate of loss of liquid from the hopper is 5 cubic centimeters per second. The height of the hopper is 30 centimeters and its diameter at the

top is 12 centimeters. Find the velocity at which the height of the liquid in the hopper is dropping at the instant when that height is 15 centimeters.