Magooey's Math Problems

Formulas for $\cos(2x)$

Synopsis. There are a number of formulas for $\cos(2x)$. It is easy to remember one of these formulas, and then generate the others by addition or subtraction. Since we all know that $\cos^2(x) + \sin^2(x) = 1$ from long experience, the process is not difficult.

Start with the formula $\cos(2x) = \cos^2(x) - \sin^2(x)$. Then add or subtract 1 from both sides.

$$\cos(2x) = \cos^2(x) - \sin^2(x) \tag{1}$$

$$1 = \cos^2(x) + \sin^2(x) \tag{2}$$

By adding equations (1) and (2) we get the formula,

$$\begin{aligned} \cos(2x) + 1 &= 2\cos^2(x) \\ \cos(2x) &= 2\cos^2(x) - 1\,. \end{aligned}$$

Similarly, by subtracting formula (2) from (1) we find

$$\begin{aligned} \cos(2x) - 1 &= -2\sin^2(x) \\ \cos(2x) &= 1 - 2\sin^2(x)\,. \end{aligned}$$

Thus from one easily remembered formula, we can generate all the possibilities.